

FAILURE OF SOLIDS UNDER STRAIN

D. A. Kiyalbaev and A. I. Chudnovskii

It is now generally accepted that failure is a process which begins virtually at the instant of load application to a specimen. Under a specific level of external forces this process is in its initial stage of a quasi-stationary character, and this makes it possible to consider it as an irreversible process and to apply to it methods of thermodynamics.

From the point of view of molecular structure the process of failure can be regarded as the development of structural irregularities, i.e., breakdown of the initial order as determined by internal forces between elements of a system and the conditions of its formation. The process is thus intimately related to changes of the system entropy. The same conclusion can be arrived at by considering the similarity of the melting and failure processes noted by Born [1, 2].

The condition of local failure is derived and a phenomenological description of failure is presented, making it possible to take into consideration the effect of various physicochemical factors. Examples of specific viscous failures are examined, and experimental and theoretical data are compared.

1. The analysis of similarity of failure and melting, and of the known empirical laws of melting [3], make possible the following assumption: A certain critical value of entropy density s^* , which is a property of material,

$$s(t^*) = s^* = s(0) + \Delta s^* \quad (1.1)$$

corresponds to a local failure.

Here t^* is the instant of failure occurrence (in the following the asterisk denotes parameters related to the state of failure), $s(0)$ is the density of entropy at the initial instant of time, and Δs^* is the critical increment of entropy density relative to the given value of $s(0)$.

Equation (1.1) may be conveniently written in the form

$$\int_0^{t^*} \dot{s}(\tau) d\tau = \Delta s^* \quad (1.2)$$

The rate of density variation of entropy can be considered as the sum of the external flux s_e' and of the internal source of entropy increase η

$$\dot{s}(t) = s_e'(t) + \eta(t) \quad (1.3)$$

In conformity with the representation of irreversible processes in thermodynamics, we write the expression for the internal source of entropy increase in the form of bilinear thermodynamic fluxes and forces

$$\eta = \sum_{\alpha} J_{\alpha} X_{\alpha} \quad (1.4)$$

As the thermodynamic fluxes J_{α} and related forces X_{α} we can choose, for example, the rate of chemical reactions and the chemical affinity, the stream of matter and the concentration gradient, the tensor of the irreversible strain rate and the stress tensor, etc.

From (1.2), (1.3), and (1.4) we obtain the condition of failure of the form

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$$\int_0^{t^*} \left[s_e^*(\tau) + \sum_{\alpha} J_{\alpha}(\tau) X_{\alpha}(\tau) \right] d\tau = \Delta s^* \quad (1.5)$$

Equation (1.5) takes into account the experimentally established fact of dependence of the failure process not only on the state of stress but, also, on various nonmechanical factors such as chemical transformations, diffusion within the material, temperature variation, and so on, i.e., failure is considered as a thermodynamic process and not as a (purely) mechanical one.

2. Let us consider the viscous failure of metal on the basis of the above premises. For simplicity we shall confine our analysis to the known Hoff's problem of failure of a metal rod of length l_0 and cross section F_0 in creep under constant tensile force P . The material is considered to be incompressible, i.e.,

$$\sigma(t) = \sigma_0 [1 + \varepsilon^p(t)] \quad (2.1)$$

and subject to the law of [creep] flow

$$\varepsilon^{p*}(t) = B\sigma^m(t); \quad \sigma(t) = \frac{P}{F(t)}, \quad \sigma_0 = \frac{P}{F_0}, \quad \varepsilon^p = \frac{l(t) - l_0}{l_0} \quad (2.2)$$

Here the dot in the superscript denotes a derivative with respect to time, and B and m , which are properties of the material, are the coefficient and the exponent of creep (stress), respectively.

The experiments whose results are compared with theoretical data are, as a rule, carried out under isothermal conditions, which implies a nonzero external entropy flux s_e^* . However, for simplicity we assume $s_e^* = 0$, since for stabilized creep this assumption yields the same results as for the isothermal case, except that Δs^* has a somewhat different value. The external entropy flux s_e^* depends essentially on the rate of load application, and, as shown by experiments described in [4], a $4 \cdot 10^3$ times increase of the strain rate results only in a fourfold increase of the work of rupturing (the specimen). This shows that under conditions close to experimental [4], the variation of the value of critical entropy is only slightly affected, when s_e^* is taken into account.

Hence the condition of failure (1.5) is written in the form

$$\int_0^{t^*} \sum_{\alpha} J_{\alpha} X_{\alpha} d\tau = \Delta s^* \quad (2.3)$$

In this problem the dissipation of mechanical energy is the source of entropy increase. The change of the total strain work density is determined by the convolution of the strain rate tensor T_{ε}^* and of the stress tensor T_{σ}

$$A^* = T_{\varepsilon}^* : T_{\sigma}$$

The strain rate tensor can be taken to be the sum of a perfectly elastic part T_{ε}^{e*} and an irreversible part T_{ε}^{p*} :

$$T_{\varepsilon}^* = T_{\varepsilon}^{e*} + T_{\varepsilon}^{p*} \quad (2.4)$$

Since a perfectly elastic deformation does not increase the medium entropy (see, e.g., [5]), the internal source of entropy, in the absence of any other factors, depends only on the rate of the irreversible strain.

For uniaxial stressing we assume

$$\eta(t) = \frac{1}{\theta} \varepsilon^{p*}(t) \sigma(t) \quad (2.5)$$

(θ is the absolute temperature)

For an isothermal process, in accordance with (2.3), we obtain

$$\Delta s^* = \frac{1}{\theta} \int_0^{t^*} \varepsilon^{p*}(\tau) \sigma(\tau) d\tau \quad (2.6)$$

Using relationship (2.1), we integrate the right-hand side of equality (2.6) and obtain

$$\Delta s^* = \frac{\sigma_0}{2\theta} [(1 + \varepsilon_*^p)^2 - 1] \quad (2.7)$$

where ε_*^p is the irreversible strain accumulated up to the instant of failure. Hence

$$1 + \varepsilon_*^p = (1 + 2\Delta s^* \theta / \sigma_0)^{1/2} \quad (2.8)$$

The integration of relationship (2.2) with (2.1) taken into account yields

$$1 + \varepsilon^p(t) = [1 - (m-1) B \sigma_0^m t]^{-1/(m-1)} \quad (2.9)$$

The latter (relationship) is valid for any arbitrary instant of time t , and in particular for t^* .

Hence the equality

$$[1 - (m-1) B \sigma_0^m t^*]^{-1/(m-1)} = \left(1 + \frac{2\Delta s^* \theta}{\sigma_0}\right)^{1/2} \quad (2.10)$$

From which

$$t^* = \frac{1}{(m-1) B \sigma_0^m} \left[1 - \left(1 + \frac{2\Delta s^* \theta}{\sigma_0}\right)^{-(m-1)/2}\right] \quad (2.11)$$

Here Δs^* is a macroscopic characteristic which is to be determined experimentally. The analysis of experimental data shows that $2 \Delta s^* \theta / \sigma_0 \ll 1$, hence it is possible in the expansion of expression

$$(1 + 2\Delta s^* \theta / \sigma_0)^{-(m-1)/2}$$

into series to retain only the first two terms. For the (length of) time up to failure we now obtain the simple relationship

$$t^* = \frac{\Delta s^* \theta}{B \sigma_0^{m+1}} \quad (2.12)$$

Formula (2.12) superficially resembles that of Hoff [6]

$$t^* = \frac{1}{m B \sigma_0^m} \quad (2.13)$$

It differs, however, from the latter in that it contains the additional characteristic Δs^* of the material which results in a better correlation with experimental data.

Experimentally established times to failure taken from [7] and those calculated by Hoff's and the author's equations, denoted, respectively, by $t_{(2.13)}^*$ and $t_{(2.12)}^*$ for tensile tests at 700°C on 1Cr13Ni16Nb steel at various σ_0 in kgf/cm², are tabulated below:

σ_0	t^*	$t_{(2.13)}^*$	$t_{(2.12)}^*$
850	2300	5932	2740
890	2170	4711	2040
965	1287	3146	1258
970	1148	3064	1230
1000	1146	2632	1020
1000	1408	2632	1020
1080	1084	1791	638
1115	706	1528	527
1200	462	1057	341
1300	292	709	211
1300	322	709	211
1305	396	695	205
1410	200	472	130
1480	175	370	98

3. This method is also applicable in the case of complex stressing. On the same assumptions as made in the preceding section, the internal source of entropy increase can be expressed in the form

$$\eta(t) = \theta^{-1} D_{\varepsilon}^{p*} : D_{\sigma} \quad (3.1)$$

Here D_{ε}^{p*} and D_{σ} are deviators of the irreversible strain and stress rates, respectively.

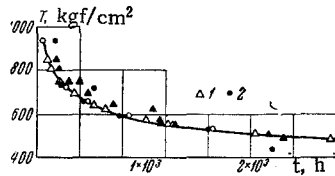


Fig. 1

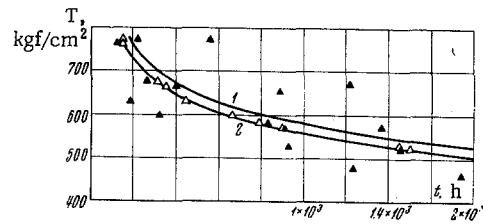


Fig. 2

Let us assume the (creep) flow to be subject to the law (see [8])

$$D_{\epsilon} v^* = 1/2 B_1 T^{m-1} D_{\sigma} \quad (3.2)$$

Here $T = (1/2 D_{\sigma} : D_{\sigma})^{1/2}$ is the shear stress intensity and $B_1 = 3^{(m+1)/2} B$. Hence

$$\eta = \theta^{-1} B_1 T^{m+1} \quad (3.3)$$

Neglecting the external flux of entropy s_e^* , we obtain for the complex stress state the following conditions of failure:

$$\Delta s^* = \int_0^{t^*} \theta^{-1} B_1 T^{m+1}(\tau) d\tau \quad (3.4)$$

For constant stress and temperature levels we derive

$$t^* = \frac{\Delta s^* \theta}{B_1 T^{m+1}} \quad (3.5)$$

As stated previously, the critical entropy increment Δs^* , a characteristic of the material (at a given initial state), is considered to be independent of the kind of stressing.

Thus the critical value of entropy in the case of complex stressing must be the same as in that of simple tension.

The last statement can be proved experimentally. Results of tests to destruction of 1Cr13Ni16Nb steel at a temperature of 700°C under various conditions of stressing are given in [7].

Some of the calculated and experimental data for this steel at temperature $\theta = 700^\circ\text{C}$ are shown in Figs. 1 and 2. In Fig. 1 the triangles relate to tensile tests, and circles to those under combined compression and torsion. Figure 2 shows curves related to the failure of a tube under pressure with curve 1 for $\Delta s^* = 1.875 \cdot 10^{-3} \text{ cal} \cdot \text{cm}^{-3} \cdot \text{deg}^{-1}$ and curve 2 for $\Delta s^* = 1.515 \cdot 10^{-3} \text{ cal} \cdot \text{cm}^{-3} \cdot \text{deg}^{-1}$.

The value of Δs^* was chosen so as to provide the best correlation between formula (2.12) and experimental data on simple tension (see tabulation), which for this steel was found to be $\Delta s^* = 1.875 \cdot 10^{-3} \text{ cal} \cdot \text{cm}^{-3} \cdot \text{deg}^{-1}$. We shall use this value for determining the time to failure of a tube under combined torsion and internal pressure by Eq. (3.5) (see Fig. 1). (Note that torsion and complex stressing are not amenable to calculation by Hoff's method [6]).

Let us apply Eq. (3.5) for determining the (time to) failure of a tube under pressure (Fig. 2). Calculations by this equation show that it yields an overestimate of the time to failure. A similar result of calculations by Hoff's equation (2.13) had been noted earlier. To explain this Kachanov [9] had introduced the concept of material embrittlement. The certain discrepancies between theory and experiment in this case appear to be of the same origin. The effect of embrittlement can be taken into account in the proposed method by the introduction of related (parameters of) thermodynamic flux and of its associated embrittlement force.*

In the examples considered above the relationship (2.6), which formally coincides with the criterion of creep proposed in [10, 11], i.e., that of specific energy dissipation in creep, was taken as the condition of failure. However the criterion of failure – the critical entropy level – proposed here differs fundamentally from that suggested in [10, 11].

* This approach was considered in Chudnovskii's paper, "Certain problems of failure of solids under strain," presented at the Seminar at the Leningrad Polytechnical Institute, Inzh. Zh., MTT, No. 5, 1969.

It should be noted that the dependence of the specific work of failure, first, on temperature and, second, on the rate of creep was established by experiments described in [4]. Thus it was found that the specific work of failure of the ÉI-10 steel at various but equal creep rates is 2-2.5 times greater at 550°C than at 650°C. As already mentioned, the effect of the creep rate on the work of failure is insignificant. These experimentally established facts can be explained as follows.

1) At a fixed critical level of entropy $s^* = \text{const}$ the work of failure – the cause of entropy increase from $s(0)$ to s^* – is the smaller the higher the input level of the system entropy $s(0)$. The latter is materially affected by the temperature.

2) The effect of the strain rate on the work of failure is related to the external flux of entropy. With increasing strain rate a more intensive heating takes place, resulting in an increase of $|s_e|$ and, consequently, in higher losses of energy dissipated during creep prior to bringing the entropy to its critical level. Furthermore, at high creep rates (the effects of) various failure mechanisms become more prominent, making it necessary to take into account in the analysis the related thermodynamic fluxes and forces.

The effect of various physicochemical factors was considered in [3] within the scope of the proposed method on the example of the effect of diffusion on failure.

LITERATURE CITED

1. M. Born, "Thermodynamics of crystals and melting," *J. Chem. Phys.*, 7, No. 8 (1939).
2. M. Born, "On the stability of crystal lattices," pt. 1, *Proc. Cambridge Philos. Soc.*, 36 (1940), pp. 160-172.
3. D. A. Kiyalbaev and A. I. Chudnovskii, "On the nature of failure," *Mechanics of lattice systems and continuous media*, Collection: *Tr. Leningr. Inzh.-Stroit. In-ta*, No. 60 (1969) [Trans. of the Leningrad Civil Engineering Inst.].
4. A. V. Stanyukovich, *Brittleness and Plasticity of Heat-Resisting Materials* [in Russian], Metallurgiya, Moscow (1967).
5. A. A. Vakulenko, "On the stress-strain relationship in elastic media, Investigations of Elasticity and Plasticity [in Russian], Collection 1, Izd-vo, LGU (1961) [Publ. by the Leningrad State University].
6. N. J. Hoff, "The necking and the rupture of rods subjected to constant tensile loads," *J. Appl. Mech.*, 20, No. 1 (1953), p. 105.
7. B. V. Zver'kov, "Long-term strength of tubes under complex loading," *Teploenergetika*, No. 3 (1958).
8. L. M. Kachanov, *Theory of Creep* [in Russian], Fizmatgiz, Moscow (1960).
9. L. M. Kachanov, "On the time to failure under conditions of creep," *Izv. AN SSSR, OTN*, No. 8 (1958).
10. V. S. Ivanova, I. A. Oding, and Z. G. Fridman, "Certain aspects of long-term strength," *Izv. AN SSSR, OTN, Metallurgiya i Toplivo*, No. 5 (1960).
11. A. F. Nikitenko and O. V. Sosnin, "Fracture in creep," *PMTF (J. Appl. Mech. and Tech. Phys.)*, No. 3 (1967).